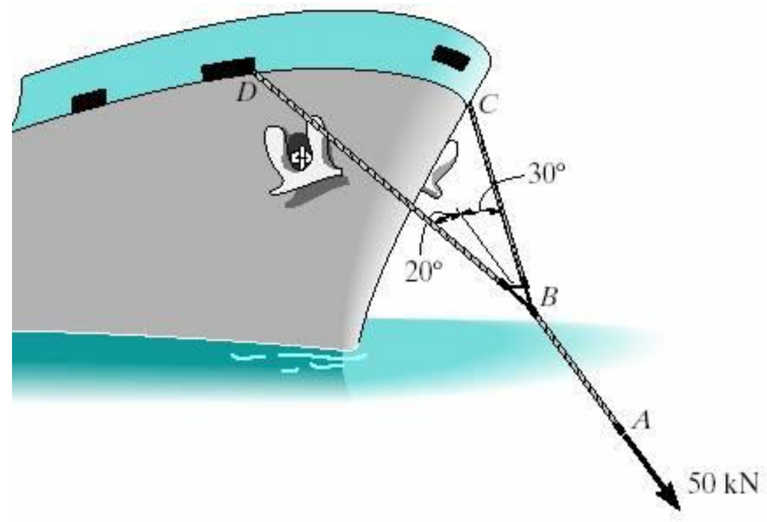
Chapter 3: Equilibrium of a particle

Applications

For a spool of given weight, how would you find the forces in cables AB and AC? If designing a spreader bar (BC) like this one, you need to know the forces to make sure the rigging (A) doesn't fail.



For a given force exerted on the boat's towing pendant, what are the forces in the cables BC and BD? What size of cable must you use?

Equilibrium of a particle

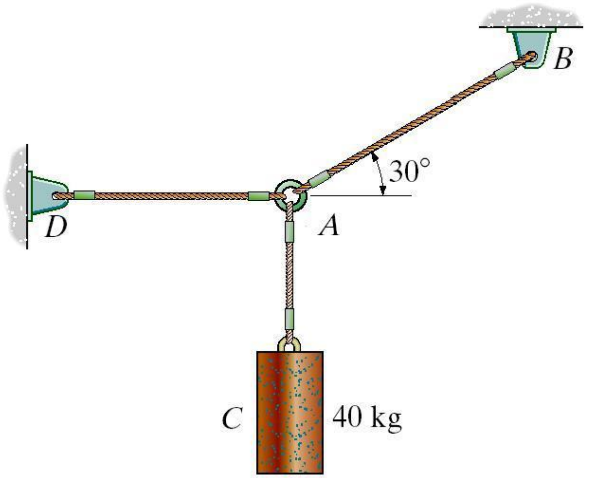
According to Newton's first law of motion , a particle will be in **equilibrium** (that is, it will remain at rest or continue to move with constant velocity) if and only if

$$\sum \boldsymbol{F} = \mathbf{0}$$

where $\sum \boldsymbol{F} = \mathbf{0}$ is the resultant force vector of all forces acting on a particle.

In three dimensions, equilibrium requires:

Coplanar forces: if all forces are acting in a single plane, such as the "xy" plane, then the equilibrium condition becomes



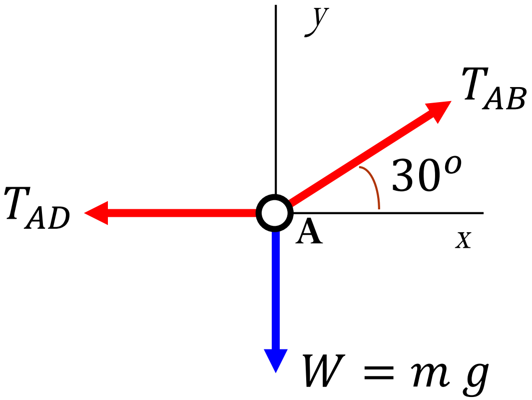
This is an example of a 2-D or coplanar force system.

If the whole assembly is in equilibrium, then particle A is also in equilibrium.

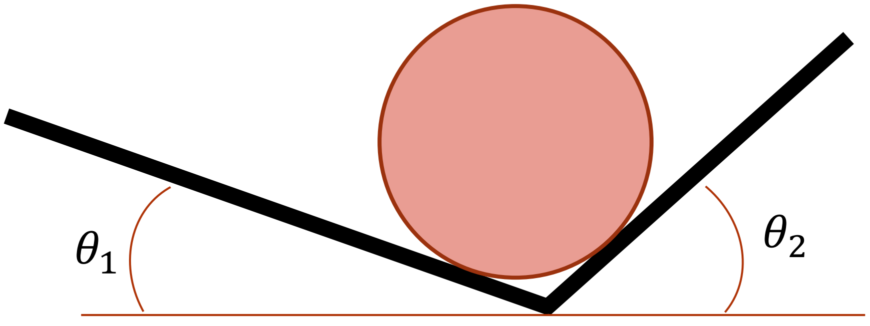
To determine the tensions in the cables for a given weight of cylinder, you need to learn how to draw a free body diagram and apply the equations of equilibrium.

Free body diagram

Drawing of a body, or part of a body, on which all the forces acting on the body are shown.



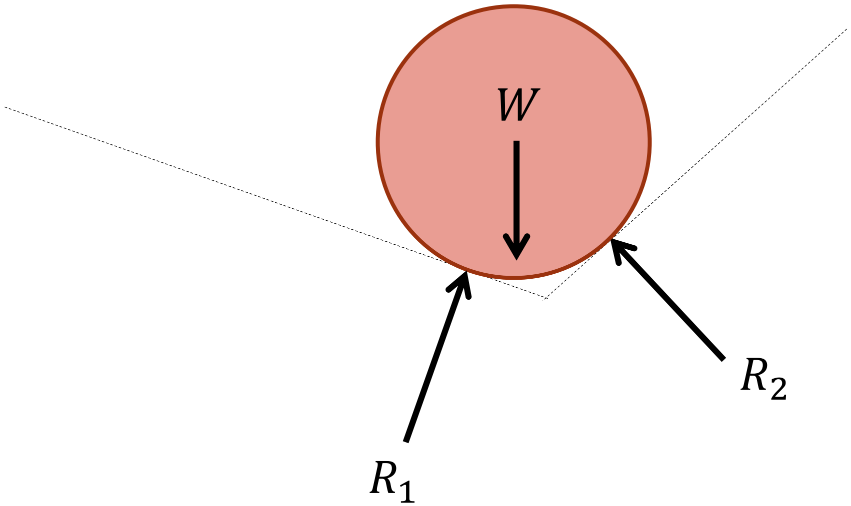
Equilibrium of a particle (cont.)



Contact force in smooth surface:

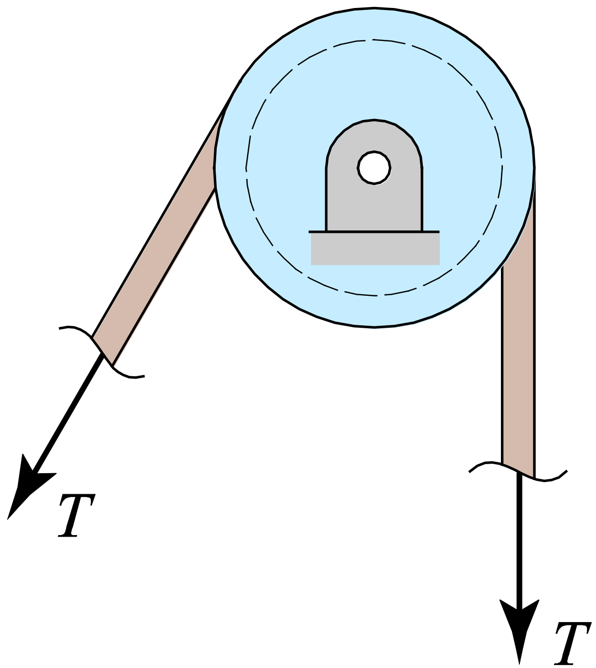
Consider now the uniform sphere of weight W, supported by smooth (frictionless) surfaces. Because the contact surfaces are smooth, the forces exerted on the sphere by the planes must be perpendicular to the surface.

Free-body diagram

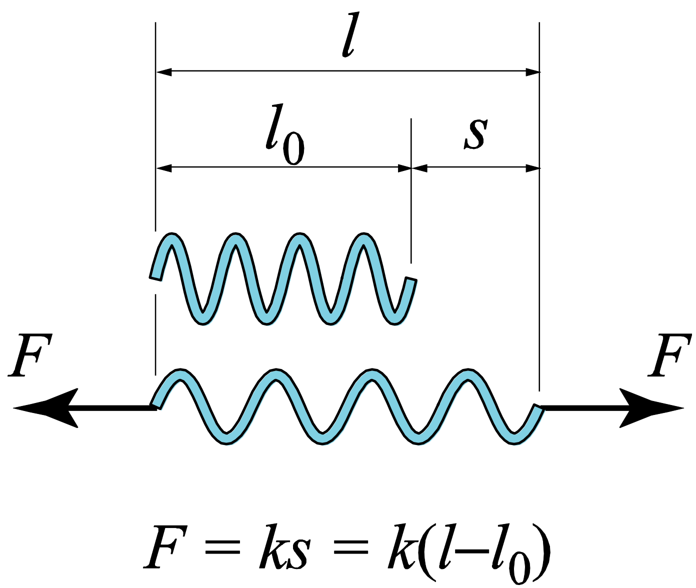


Idealizations

Pulleys are (usually) regarded as frictionless; then the tension in a rope or cord around the pulley is the same on either side. Springs are (usually) regarded as linearly elastic; then the tension is proportional to the *change* in length *s*.

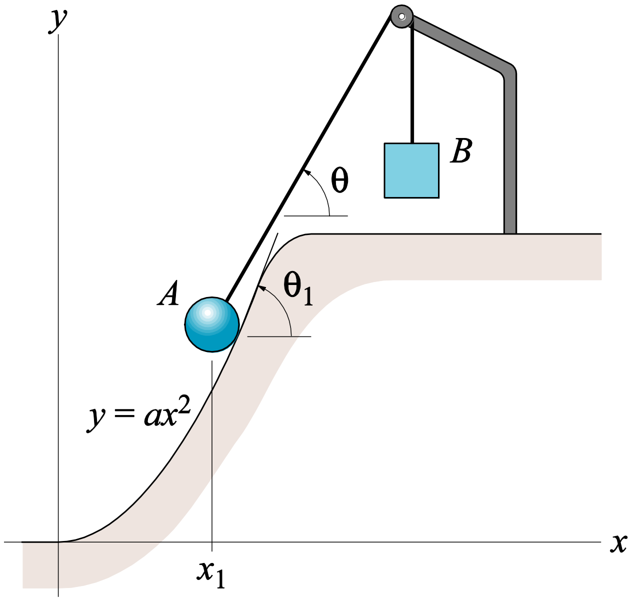


Frictionless pulley

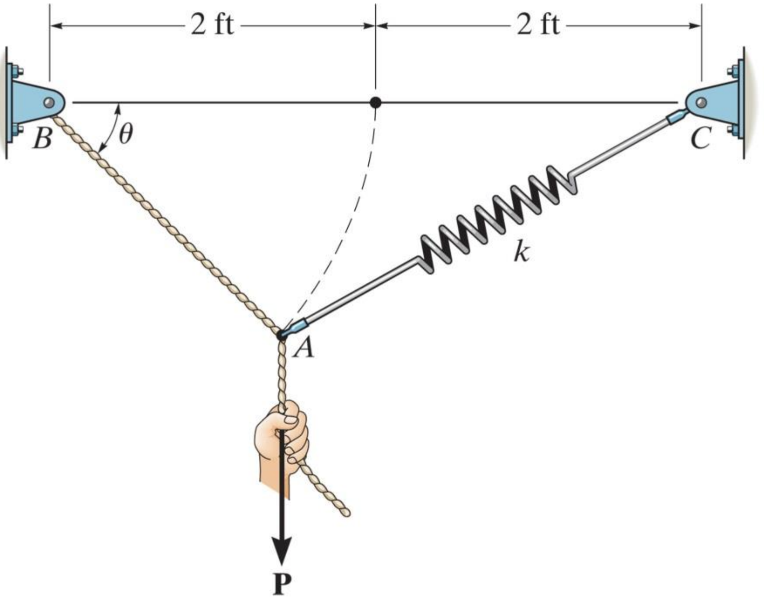


Linearly elastic spring

A 4 kg sphere rests on the smooth parabolic surface. Determine the normal force it exerts on the surface and the mass m_B of block B needed to hold it in the equilibrium position shown. The given parameters are:
$x_1 = 0.4$ m, $a = 2.5$ m^{-1}, $\theta = 60^o$.



Determine the unstretched length of spring AC if a force P = 80 lb causes the angle $\theta = 60^o$ for equilibrium. Cord AB is 2 ft long. Use the spring stiffness k=50 lb/ft.

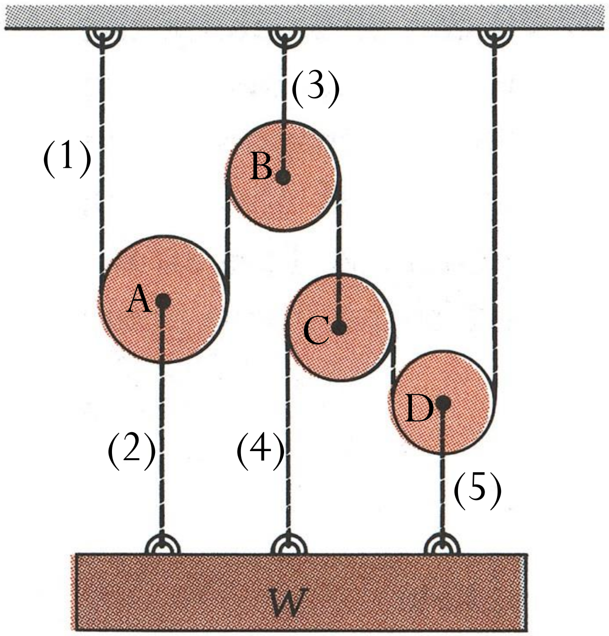


Equilibrium of a system of particles

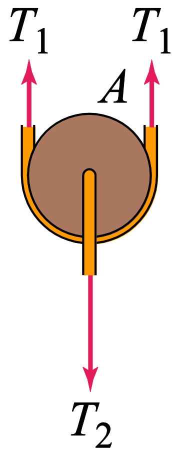
Some practical engineering problems involve the statics of interacting or interconnected particles. To solve them, we use Newton's first law

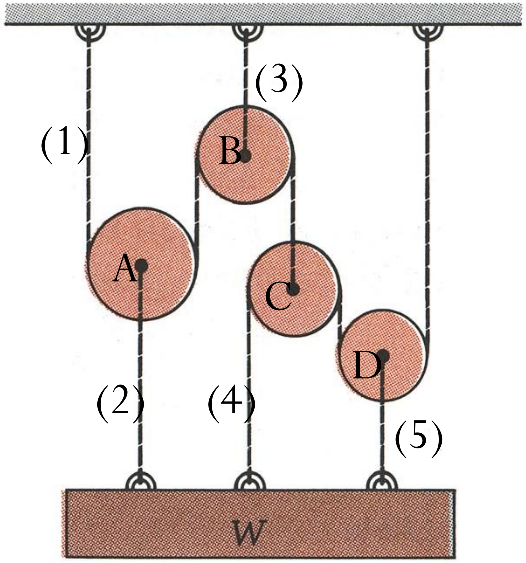
$$\Sigma \mathbf{F} = \mathbf{0}$$

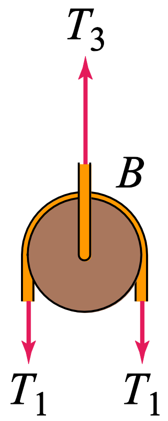
on selected multiple free-body diagrams of particles or groups of particles.

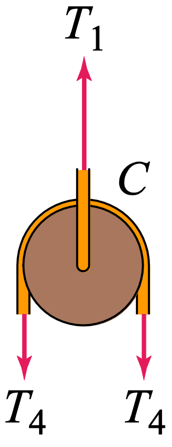


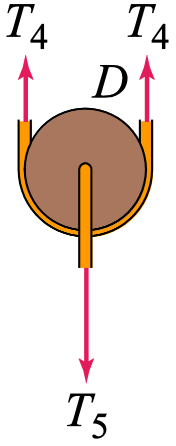
The five ropes can each take 1500 N without breaking. How heavy can W be without breaking any?



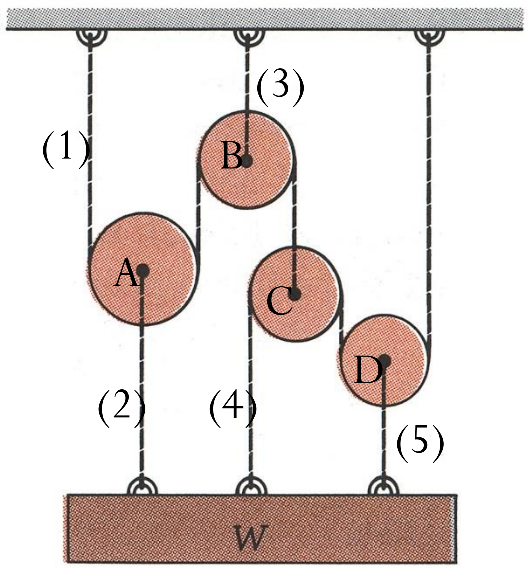

(1)
(2)
(3)
(4)
(5)
A
B
C
D
W


T_3
B
T_1
T_1

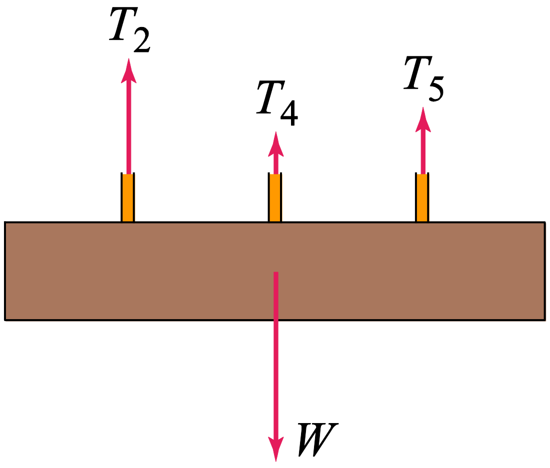

T_1
C
T_4
T_4


T_4
T_4
D
T_5

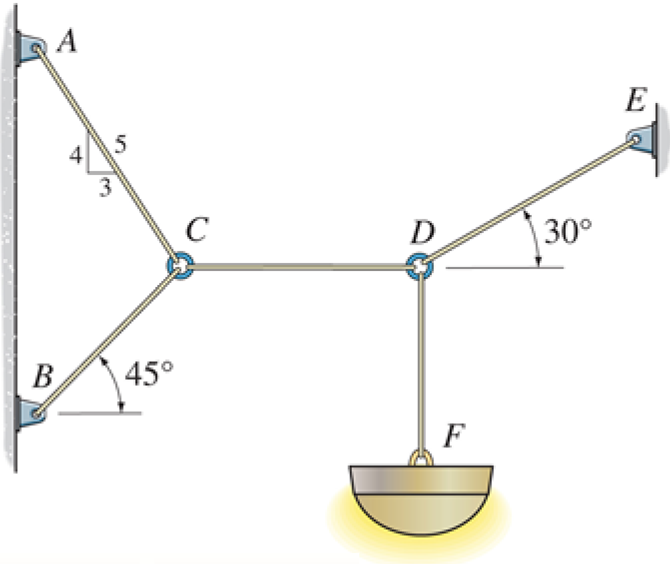
Therefore, the critical tension is reached simultaneously in ropes (2) and (3)

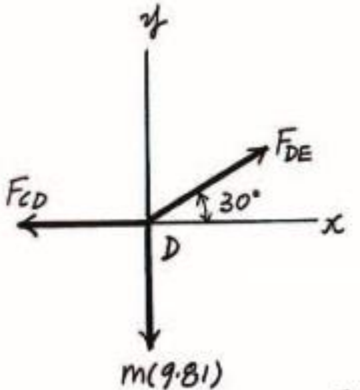


Free-body diagram of the weight:

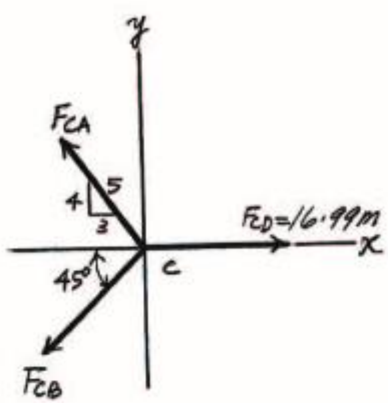


Determine the maximum mass of the lamp that the cord system can support so that no single cord develops a tension exceeding 400N.





(a)



(b)

3D force systems

This shear leg derrick is to be designed to lift a maximum of M=200 kg of fish. Find the magnitude of the forces acting in the cable and derrick legs? Use $a = h = 4$ m. What happens to these forces when the offset distance decreases, i.e., during the lifting of the fish net until the legs are at a perpendicular position?